



The Economics of Mangrove Forest Use and Management

Tai Shzee Yew

Faculty of Economics and Management and
Institute of Agricultural and Food Policy Studies
Universiti Putra Malaysia
43300 UPM, Serdang, Selangor, Malaysia

➤ Main objective

- Introduce participants to the basic economics of (mangrove) forest utilization and management

Part 1:

The basic economic principles

- Scarcity, choice and opportunity cost
- Marginal values and economic decisions
- Incentives and substitution
- Property rights and natural resource use
- Decision making over time

Part 2:

The Economics of Forest Use

- Some biological Basics of Trees and Forests
- The Efficient Rotation Period for the Firm
- Management Policies

PART 1: THE BASIC ECONOMIC PRINCIPLES

- The basic economic principles
- Scarcity, choice and opportunity cost
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Scarcity, Choice and Opportunity Cost

- Scarcity is a relative concept: comparison of wants (or objectives) and resources (or means).
- Scarcity arises whenever wants exceed resources.
- Resources are capable of satisfying human wants, are also defined in relation to wants.
- E.g.: - Petroleum stock in the ground - not a scarce resource before the age of combustion engines.
 - Uranium.
 - Air.

- Problem of scarcity leads to choice has to be made.

E.g.: Petroleum.

Atmosphere.

Time.

- Choosing something means giving up something else – the principle of opportunity costs.
- Opportunity cost is the cost expressed in terms of the next best alternative foregone.

E.g.: The costs of going to school involve not only the tuition fees paid, it should also include the living expenses (paid item) and the foregone earnings (unpaid item).

- When costs are seen in this way, nothing is really costless (or free), as there are always alternatives to our choice.
- Thus, economics can be defined as the science of choice and can be found everywhere.

A note on resources:

- Resources in economics can be classified into 3 main classes – land, labour and capital.
- Land is the general term for resources found in nature.
- Labour refers to the human physical and mental effort involved in production.
- Capital is man made capital like machines and equipments.

Marginal Values and Economic Decisions

- Marginalism is a fundamental idea in economics. It proposes that choices are made in small incremental steps – or at the margin.
- Decisions are made by comparison between additional costs (marginal costs) to be incurred and the additional benefits (marginal benefits) to be received.

E.g.: Consumers making decisions on goods purchased for consumption – demand schedule.

Producers or firms making decisions on goods supplied to the market – supply schedule.

Producers making decision on how much of an input to use.

(a) The Demand Schedule

Price	Quantity
1	9
2	6
3	4
4	3
5	2

(b) The Demand Curve

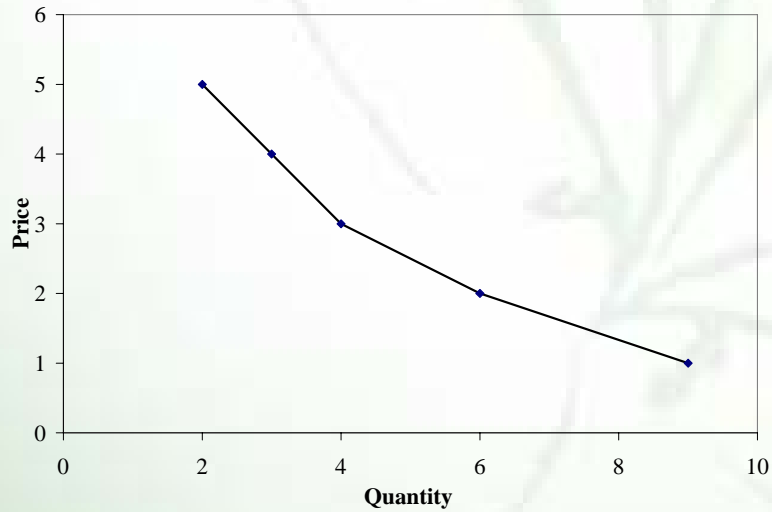


Figure 2.1: The Demand Schedule and Curve

(a) The Supply Schedule

Price	Quantity
1	0
2	3
3	4
4	5
5	6

(b) The Supply Curve

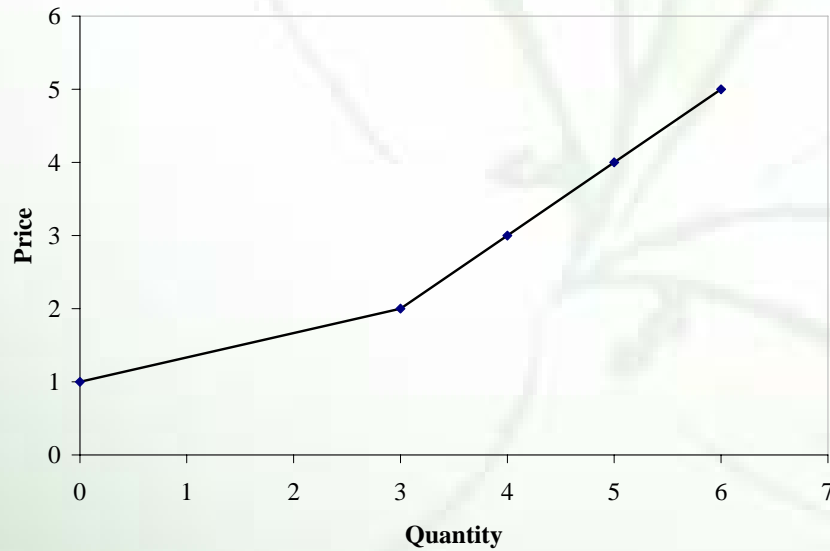


Figure 2.2: The Supply Schedule and Curve

Incentives and Substitutions

- Substitution is another fundamental principle in economics.
- Changes in opportunity costs and marginal benefits will render previous decisions obsolete.
- Substitutions between higher opportunity cost choices with lower opportunity cost choices and substitutions between lower marginal benefits with higher marginal benefits are constantly being made.
- Thus, lowering opportunity costs and increasing benefits are powerful incentives for actions.
- Offerings of incentives for substitutions lead to competition.

Property Rights and Natural Resource Use

- A property right is a bundle of characteristics that convey certain powers to the owner of the right.
- These characteristics include: exclusivity, transferability, enforceability, divisibility and duration.
- Exclusivity is an important characteristic of property right in relation to natural resource use.
- With exclusive right, the owner of the natural resource:
 1. Can prevent others from using the natural resource without permission;
 2. All benefits and costs from using the resource should accrue only to the owner.

Exclusive right can be granted to:

- Private individual in the form of private property right
- A group of individuals in the form of common property right
- None, leading to open access
- With private property rights, markets for the natural resources exist, and allocation of resources is efficient without government intervention.
- Open access resource, on the other hand cannot achieve efficient allocation of resources without some form of government intervention, creation of private property right or both
- For mangrove forest utilization:
 - Exclusive right to the trees is usually granted to private individual
 - However, the right to other resources such as fisheries, habitats for animal, aesthetic environment, and others remains open access.

Externality

- Exists because of absence of well defined property right.
- Exists whenever the welfare (benefits or costs) of an agent (firm) depends not only on its own activities, but also on activities under the control of some other agents.
- E.g.: Clearing of mangrove swamps for commercial shrimp farming in Tha Po village in Thailand (Suthawan and Barbier, 2001)
 - o Tha Po villages experiences decline in fish catch, suffered storm damage and water pollution.
 - o These costs are not borne by the shrimp farmers but by the Tha Po villagers, hence they represent the external costs to the villagers.
 - o Thus, private costs (the costs of production inputs) of the shrimp farmers may differ from the true (social) costs by the amount of the external costs involved.
- Note also that benefits not remunerated to the producers or unpaid for by the benefactors are the external benefits.

Discounting and Present Value

- Are based on the idea of time preference where people prefer to realize benefits sooner rather than later (and realize costs later rather than sooner).
- A dollar today is not indifferent to a dollar in the future.
- Discounting is a procedure of expressing benefits in different periods to the present value.
- In discounting, all future values are converted to a value in today's dollars with the conversion constructed in such a way that individual is indifferent between the dollars in the future and the present value of those dollars today.
- E.g.: suppose you have been given an offer to buy a bond for \$100 today and you have been told that the bond would pay you \$110 one year from now. Are you willing to buy it today?
- If your answer is yes, you are indifferent between \$100 today and \$110 one year in the future or you view the present value of \$110 one year from now to be \$100.
- Your rate of time preference or your discount rate is 10 percent.

- The formula for obtaining the present value is given as:

$$PV = (1+r)^{-t} (FV) \quad (2.1)$$

where PV refers to the present value, t is the time period, r is the discount rate, and FV is the future value.

- E.g.: Calculate the present value of \$1000 payable in 10 years at a discount rate of 8 percent.
- In resource economics, a slightly more complex formulation of deriving the present value is necessary because resources typically generate benefits and costs each year for over a period of a number of years.
- If we let B_t to represent the benefits for year t, C_t be the corresponding costs in each year, and T be the number of years in which the resource will be yielding either costs or benefits, the present value of the total net benefits are given by :

$$TPV \text{ of NB} = \sum_{t=1}^T (B_t - C_t)(1+r)^{-t} \quad (2.2)$$

The Economics of Mangrove Forest Use

- Mangrove forest is a renewable resource.
- The growth of the forest relies on natural regeneration and replanting to produce new generations of trees.
- Mangrove forest provides a sustained flow of marketable products and services.
- E.g. for the Matang mangrove forest, timber (poles), timber related products (charcoal), and fish constitute the main output produced, while providing nursery grounds for fisheries, carbon sequestration, and storm and soil protection are other services of the mangrove forest.

- This section focuses on the optimal use of mangrove forest resources.

- The topics to be discussed include:
 - o Timber harvest based on the biology of trees
 - o Optimal timing of timber harvest on initially bare land.
 - o Efficient harvests in a mature forest.
 - o Non-timber values and optimal timing of timber harvest.
 - o Public policy instruments and their impact on mangrove forest management.

Timber Harvest Based on the Biology of Trees

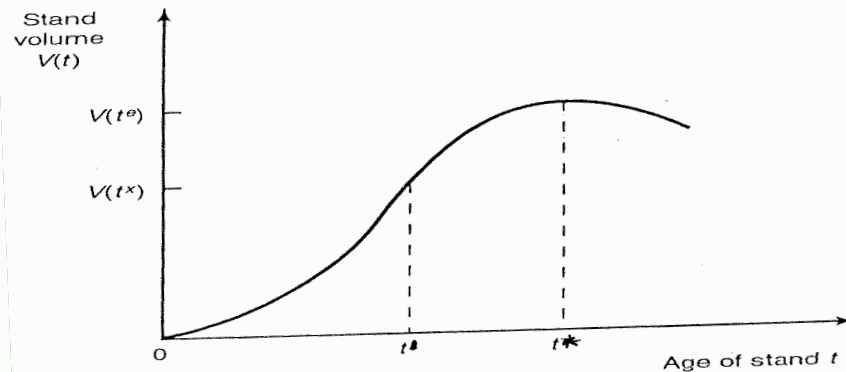


Figure 3.1: The growth cycle for a representative forest stand with trees of identical age. The maximum wood volume is obtained when the stand is t^* years old. Beyond t^1 , the growth rate slows.

- The amount of wood usable for commercial harvests changes over time as tree grows .
- Wood volume per unit area $V(t)$, develops slowly in the early stages and then up to age t^1 speeds up. It slows toward age t^* , when the maximum volume is achieved.

- The volume-age relationship can be rewritten in terms of the annual increase in volume schedule in Figure 3.2.

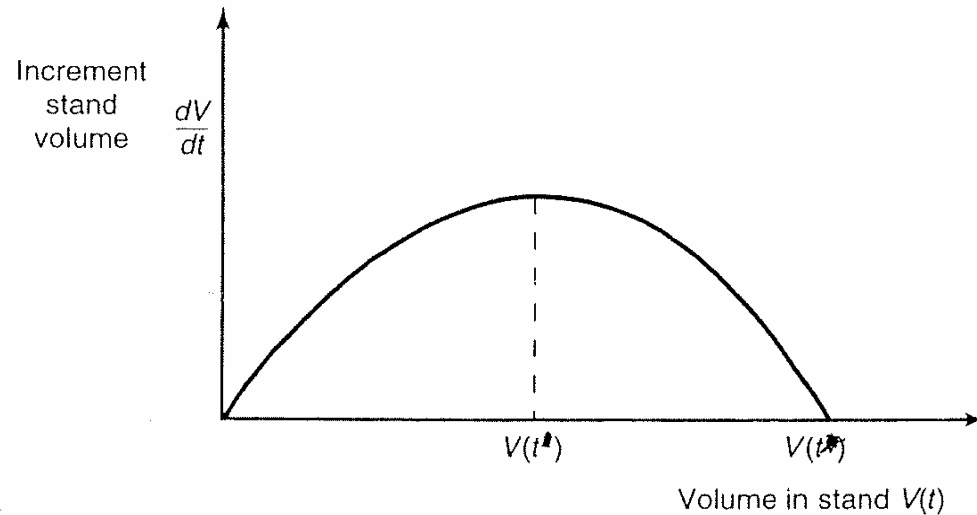


Figure 3.2: This schedule is derived from Figure 3.1. Instead of age, the increment in wood volume at different ages is plotted against the volume of wood in the stand of trees.

- The volume-age relationship can be altered by fertilizing, thinning trees, repressing pests and having optimal density of trees on a plot.
- What is the best time to harvest a tree or stand of trees?
- Table 3.1 shows the total and average volume of wood, and the yearly growth rate, in successive decades of the forest life
- If planning horizon is 1,000 years:
 - o Maximizing total volume at a cycle of 100 years yield about 20,900 cubic feet
 - o Maximizing average yield at a cycle of 60 years yield 27,666 cubic feet.
 - o Thus, the 60-year cycle would yield the larger volume and could be thought of as the maximum sustainable yield for the forest.

Table 3.1: Total Volume, Average Volume, and Annual Increase in Volume of Wood, by Decades, of One Acre of Hypothetical Forest

Age of trees (decade)	Total volume of wood (cu. ft)	Average volume (cu ft/age)	Annual increase in volume (cu ft/yr)
0	0	0	0
1	80	8	8
2	200	10	12
3	400	13.3	20
4	720	18	32
5	1360	27.2	44
6	1660	27.7	30
7	1840	26.3	18
8	1960	24.5	12
9	2040	22.7	8
10	2090	20.9	5
11	2090	19	0
12	2090	17.4	0
14	2090	14.9	0

- Is the harvest age at the maximum sustainable yield of the forest also maximizes net benefits to society?
- In practice, it depends on:
 - o The biology
 - o Final use of the harvested trees
 - o Whether we begin with bare land or a mature forest
 - o Cost
 - o Productivity
 - o Demand conditions
 - o Leasing or ownership arrangements.

Simple Model of the Timing of a Timber Harvest

- Assumptions:
 - A tract of land on which trees must first be planted.
 - The stand of trees has uniform age and growth characteristics and will be harvested in the future.
 - Once harvested, saplings will be replanted and the cycle begins again.
 - Harvesting and replanting occur within the same period of time – say 1 year
- Objective:
 - o To determine the economically efficient rotation period over an infinite number of cycles of planting, harvesting, replanting, and harvesting to maximize the present value of the land in growing trees.

- Direct costs involved are:
 - o Actual costs of managing a forest - costs of planting, silviculture, harvesting, storage, transportation to market, and so on
 - o The interest forgone while waiting to harvest the trees (or the opportunity cost of waiting)
- Indirect cost:
 - o The value of the land on which the trees grow
 - o Land receives the residual income – the rent, the income remaining after all planting, maintenance, and harvesting costs have been paid
- The optimal rotation interval chosen will maximize the land rent.

- Let the planting cost per unit of land (say an acre) be fixed at \$D, cost of harvest the trees at \$c per cubic foot of wood, the date of planting be T_0 and the date of harvesting be T_1 .
- The present value cost of the ‘first round’ in the infinite cycle is

$$D + cV(T_1 - T_0)e^{-r(T_1 - T_0)} \quad (3.1)$$

- Let the revenue per cubic foot of timber sold upon harvesting be constant at \$p.
- The present value profit to the owner of the forest land from planting and harvesting is

$$(p - c)V(T_1 - T_0)e^{-r(T_1 - T_0)} - D \quad (3.2)$$

- After harvesting, the land is replanted at cost D and a new round is undertaken, then a third round, and so on. The complete present value is then:

$$\begin{aligned}
 W = & [(p - c)V(T_1 - T_0)e^{-r(T_1 - T_0)} - D] \\
 & + e^{-r(T_1 - T_0)} [(p - c)V(T_2 - T_1)e^{-r(T_2 - T_1)} - D] \\
 & + e^{-r(T_2 - T_0)} [(p - c)V(T_3 - T_2)e^{-r(T_3 - T_2)} - D] \\
 & + K
 \end{aligned} \tag{3.3}$$

- Let the interval between planting and harvesting $(T_1 - T_0)$, $(T_2 - T_1)$, $(T_3 - T_2)$, and so on be the same and be denoted by I , the rotation interval. Equation (3.3) becomes:

$$\begin{aligned}
 W = & [(p - c)V(I)e^{-rI} - D] + e^{-rI} [(p - c)V(I)e^{-rI} - D] \\
 & + e^{-2rI} [(p - c)V(I)e^{-rI} - D] \\
 & + e^{-3rI} [(p - c)V(I)e^{-rI} - D] \\
 & + K
 \end{aligned} \tag{3.4}$$

- Factor out the term e^{-rI} yields

$$\begin{aligned} W = & [(p - c)V(I)e^{-rI} - D] \\ & + e^{-rI} \{ [(p - c)V(I)e^{-rI} - D] + e^{-rI} [(p - c)V(I)e^{-rI} - D] \\ & + e^{-2rI} [(p - c)V(I)e^{-rI} - D] \\ & + K \} \end{aligned} \quad (3.5)$$

- The term in bracket { } in Equation (3.5) is the same as Equation (3.4). Substituting W into Equation (3.5) gives

$$W = [(p - c)V(I)e^{-rI} - D] + e^{-rI}W$$

- Solve in terms of W, and obtain

$$W = [(p - c)V(I)e^{-rI} - D] \left(\frac{1}{1 - e^{-rI}} \right) \quad (3.6)$$

- The forest manager then maximizes W , by choosing a rotation interval I such that $dW/dI = 0$. This yields the condition

$$(p - c)V'(I) = r(p - c)V(I) + rW^* \quad (3.7)$$

- Where $V'(I)$ is the derivative of $V(I)$ with respect to I .
- The left-hand side of equation (3.7) is the value of the marginal product of the timber if it is held another period.
- The right hand side of the equation (3.7) represents two opportunity costs:
 - $r(p-c)V(I)$ is the interest on the money the landowner would earn if she or he had cut the timber in the stand and put the money into a bank to earn interest at rate r .
 - rW^* is the opportunity cost of the land – the interest earned if the land is sold for its optimal timber production value, W^* (site value).
- Site value represents the greatest amount that could be offered for the un-forested land if it is used to grow timber.
- rW^* represents the site rent.

- Figure 3.3 shows a typical relationship between W and stand age for three interest rates: 3, 5, and 8 percent. As interest rates increase, the site values at all stand ages diminish.

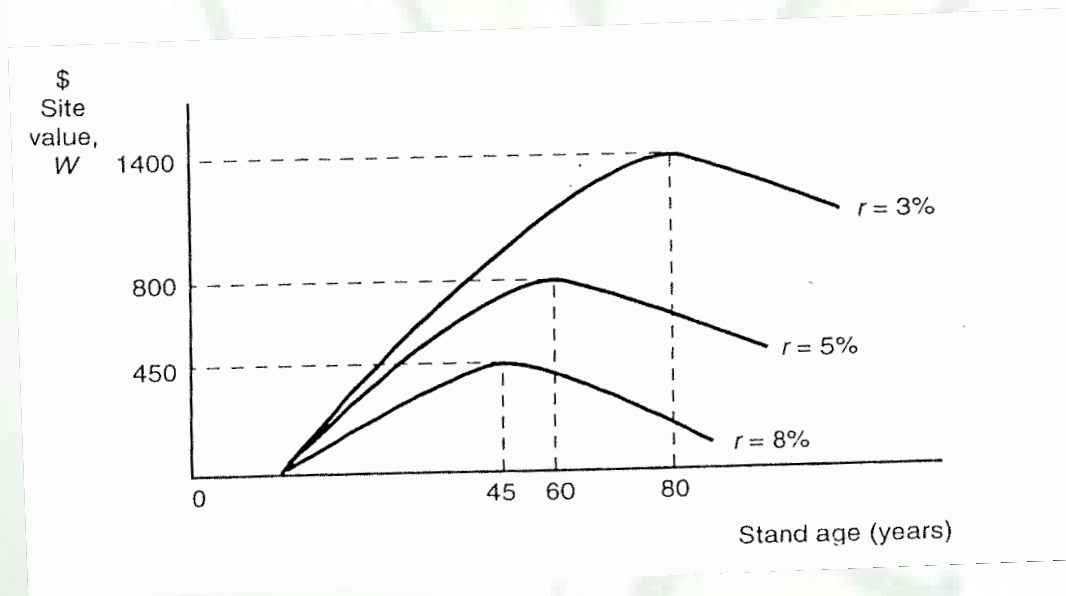


Figure 3.3: Site values for different interest rates are shown. For each interest rate, r , a unique maximum value of forested land, w , is found. Higher interest rates mean lower site values.

- The optimal rotation interval, I^* , is determined where the VMPT equals TOC as shown in Figure 3.4.
- $VMPT = (p-c)V'(I)$ represent the value of the marginal product of the forest in growing timber.
- The TOC curve represents the two opportunity costs: $r(p-c)V(I)$ plus rW^*

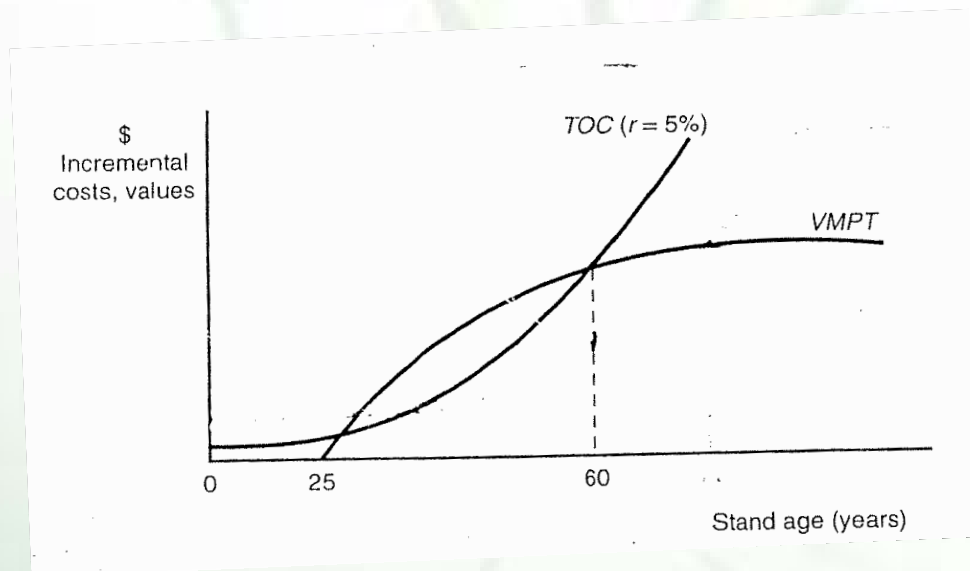


Figure 3.4: Optimal rotation interval is 60 years if $r=5\%$. At this age, the value of the marginal product of growing timber, VMPT, equals the opportunity cost of deferring the forest.

Efficient Harvests in a Mature Forest

- The first harvest involves a distinctly unplanned forest. Does this make the analysis of the efficient rotations irrelevant?
- Suppose the forest is to be clear-cut resulting in a net revenue of $\$K$ in the first harvest, and is independent of rotation time.
- This $\$K$ is a fixed value at the beginning of the problem and does not affect subsequent marginal decisions and thus we do not have to amend our model or our general concept of a planned planting and harvesting cycles.
- However, if the original forest is thinned and gradually adjusted to an efficient program with selective cutting, a very complex planning problem emerges.

Socially Optimal Rotation Intervals: Non-timber Values

- Non-timber uses include the values of the mangrove forest land as a nursery for fish, carbon sequestration, and wildlife and ecological preservation.
- Incorporating non-timber values (NTVs) affects the rotation interval.
- It may increase, decrease, or not change very much the decisions about timber harvesting.
- There are a number of difficulties with evaluating NTVs because many uses of the forest do not have well-defined market values. Thus these values need to be imputed.
- How will the efficient rotation interval might be affected by the inclusion of NTVs?

- Assumptions:
- At the beginning there is a bare land and the entire plot is planted within one time period, so that all the growing trees will be of the same age.
- The benefits from NTVs are also a function of the age of the stand denoted by $N(t)$ per acre of forested land at any stand age t .
- The present value from NTVs is:

$$\begin{aligned} B = & [N(T_1 - T_0)e^{-r(T_1 - T_0)}] \\ & + e^{-r(T_1 - T_0)} [N(T_2 - T_1)e^{-r(T_2 - T_1)}] \\ & + e^{-r(T_2 - T_0)} [N(T_3 - T_2)e^{-r(T_3 - T_2)} - D] \\ & + K \end{aligned} \tag{3.8}$$

- Assuming that all the rotation intervals will be of the same length and factor out e^{-rI} from (3.8) and substitute a B, to obtain

$$B = [N(I)e^{-rI} - D] \left(\frac{1}{1 - e^{-rI}} \right) \quad (3.9)$$

- The social planner then maximizes the total value of the forest, F, in producing timber and non-timber benefits, where $F=W+B$, with W defined from equation (3.6) and B from (3.9).
- Maximizing F with respect to I yields the condition that I must satisfy

$$(p-c)V'(I) + N'(I) = r(p-c)V(I) + rF^* \quad (3.10)$$

- where $F^*=(W^*+B^*)$ is the optimal value of the land including both timber and NTVs.
- Equation (3.10) can be interpreted as the optimal multiple use of the forest.

- The optimal rotation is at a stand age that equates the value of a marginal increase in stand age (the left-hand side of equation 3.10) with the opportunity costs of delaying the harvest – the foregone interest on the timber harvested plus the foregone land rent on delaying the rotation (the right-hand side).
- Thus NTVs enter in two ways – through $N'(I)$ and rB^* .
- There is no reason to expect $N(I)$ to be increasing or decreasing throughout its range as I increases.
- Thus the socially optimal rotation interval for multiple-use forest management may be longer than, shorter than, or virtually identical to the efficient rotation interval for timber harvests.
- It is of course also possible that if $N(I)$ is very large, there will be no rotation interval at which equation (3.10) is satisfied, i.e., the timber is never harvested.
- In general, however, we expect the socially efficient rotation interval under multiple use to be somewhere between the timber management rotation and the stand age that maximizes the present value of the returns from NTVs alone.
- If NTVs rise with stand age, the multiple-use rotation will be greater than the timber-harvest-only rotation. However, NTVs could fall with stand age, and this will lower the rotation interval.

Management Policies

- Forest management policies instituted by the government can be classified into two major categories:
 - Those that influence optimal timber rotations and
 - Those that influence property rights to the land, to timber growing on both publicly and privately held land, or to both
- They include:
 - a. Tax per ton harvested or royalty or severance tax.
 - b. Site-use tax.
 - c. Profit tax.
 - d. License fee.
 - e. Subsidy for reforestation costs.
 - f. Concessions.

- **Tax per ton harvested** is usually paid as percentage of the value of timber when harvested. It is equivalent to an increase in harvesting costs from c to $c+t$, where t is the tax.
- An increase in c leads to a longer rotation interval.
- **Site-use tax** is a tax per acre each time land is brought into forestry use. This is equivalent to an increase in the setup costs, D . The effective setup cost would become $D+T$, where T is the total site-use tax.
- The effect of an increase in D and hence the site-use tax is to increase the profit-maximizing rotation interval.
- **A tax on profits** of t percent will not change the optimal rotation interval. Since profit is the residual income ascribable to land in forest production, this “profit” tax can be viewed as a tax on the income accruing to landowners. The tax cannot be shifted by altering the rotation interval.

- **License fee per acre** is the same as an increase in the setup costs, D . This will lengthen the rotation interval.
- If the license fee is levied per year, it can be viewed as a fixed cost and will reduce the present value of profits only, but the optimal rotation interval is unaffected because the license fee is related neither to the amount harvested nor indirectly to the planting-harvesting interval.
- **Subsidy** for a portion of the reforestation costs on efficient rotation interval is identical to a reduction in D . The rotation interval will be shortened.
- **Concession** of public forest land is a major type of forest management policies that influences property rights.
- A concession is used to describe the terms under which the rights to harvest timber or manage the timber-producing land within a specific period of time.

- Several considerations in determining concessions. These include:
- Ways to issue them (through grants, bidding processes, etc.)
- Their duration
- The extent of rent capture, and other features.
- The terms of the contracts between the government and the leaseholder can affect rotation intervals, the nature of the harvesting, the treatment of non-timber values, depletion of the forest, and more.

- Two important issues with respect to concessions to harvest public forested lands:
 1. Whether or not they are sold for a price that reflects the present value of the harvests. Evidence from developed and developing countries suggests that concessions are sold or lease or bid for “below market value” or sometimes below the costs of producing timber which are not in the public interest (see Bowes and Krutilla, 1989).

2. The length of tenure.

- Leases typically last for less time than the rotation interval.
- The short-term leasing arrangements creates an incentive for firms to harvest more quickly than if they had secure tenure.
- It also discourages investment in silviculture, reforestation, and other methods of increasing the productivity of the forest.
- Firms may therefore treat the forest as if only one harvest is available, not as a sustainable resource.

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*Thank You Very Much
For Your Attention*

Selected Readings

Some of the references listed below were not explicitly cited in the text. They are mainly basic texts of varying levels of difficulty, aimed at providing those who wish to read further in this field.

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Increase in Planting Cost, D

- A rise in planting or clearing costs in the basic model implies a longer rotation cycle.
- When D rise, the value of land in forest use, W^* must decline. This implies that the second rotation and subsequent rotations are worth less because the planting and clearing costs have risen. The response should be to delay getting to the second and subsequent rotations.

$$\frac{dV(I)}{dI} = rV(I) + \frac{r}{(p-c)}W^* \quad (3.7)$$

- W^* is the optimal value of the program of planting and harvesting for the efficient rotation interval in perpetuity, and is equal to .
- Totally differentiate equation (3.7) to obtain:

$$\frac{\partial \left[\frac{dV(I)}{dI} - rV(I) \right]}{\partial I} dI = \left[\frac{r}{(p-c)} \right] \frac{\partial W^*}{\partial D} dD = \left[\frac{r}{(p-c)} \right] \left[\frac{-e^{-rI}}{e^{-rI} - 1} \right] dD$$

- Both sides are negative, so $\frac{dI}{dD} > 0$

Change in the Net Price of Timber (p-c)

- If price falls or c rises, net price decreases.
- A decrease in the net price will reduce the value of the marginal product of the timber, $(p-c)V'(I)$, as well as reduce both of the opportunity costs of delaying the harvest.
- The VMPT curve shifts down, as does the TOC. This leads to a longer rotation interval as forest owners shift their harvests to the future.

$$\frac{\partial \left[\frac{dV(I)}{dI} - rV(I) \right]}{\partial I} dI = \frac{-rW^*}{t^2} dt + \frac{r}{t} \frac{\partial W^*}{\partial t} dt = \frac{r}{t} \left\{ \frac{-tV(I) + e^{rI} D}{(e^{rI} - 1)t} + \frac{V(I)}{e^{rI} - 1} \right\} dt$$

$$= \frac{rD}{t^2} \left[\frac{e^{rI}}{e^{rI} - 1} \right] dt$$

- The left-hand side is negative and the right hand side is positive, so we have

$$\frac{dI}{dt} < 0 \text{ or } \frac{dp}{dt} < 0 \text{ and } \frac{dI}{dc} > 0$$

Increase in Interest Rates, r

- A higher r implies a shortening of the efficient rotation interval.
- The opportunity costs of holding the forest in uncut timber rise.
- Because W^* is a present value, a higher r means a lower value of the forest land for all time periods. Owner will want to harvest more quickly.
- The forgone interest on timber that could have been harvested is also higher, so the term $r(p-c)V(I)$ also increases.

$$\frac{\partial \left[\frac{dV(I)}{dI} - rV(I) \right]}{\partial I} dI - V(I)dr = \frac{W^*}{p-c} dr + \left(\frac{r}{p-c} \right) \frac{\partial W^*}{\partial r} dr$$

$$\frac{\partial \left[\frac{dV(I)}{dI} - rV(I) \right]}{\partial I} dI = \frac{W^*}{p-c} \left[1 - \frac{rI}{e^{rI} - 1} \right] dr + V(I) \left[1 - \frac{rI}{e^{rI} - 1} \right] dr$$

- The left-hand side is negative and the right-hand side is positive, so we have $\frac{dI}{dr} < 0$